Short Foucault pendulum: A way to eliminate the precession due to ellipticity

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The problem of ellipticity in the motion of the ordinary Foucault pendulum and the error caused by it are discussed. A simple method of slightly modifying the force–displacement relation in such a way that precession does not result from ellipticity is presented. The practical range of length of the Foucault pendulum is extended downward substantially. As experimental verification, results from a 70-cm pendulum, mounted as a "wall clock" are given. Rates of turning from daily readings show a 2% rms deviation from the mean, and the long-term average is within 2% of the rate calculated for the local latitude.

I. PURPOSE

The problem of the precession caused by elliptical motion rapidly becomes more serious as the length of a Foucault pendulum is decreased. For short pendulums (a few meters) the standard practice is to use a Charron ring (described later) or a similar device to limit the ellipticity to a tolerable value. This has extended the range of successful operation to less than 2 m.[1] (Attention is directed to the extensive footnotes, of which this is the first, used throughout the paper.) At shorter lengths even the small residual ellipticity causes so much precession that the observation of the Foucault effect is difficult. The purpose of this paper is to present a different solution to the problem: a method of eliminating, not the ellipticity itself, but the precession caused by it.

II. BACKGROUND OF THE PROBLEM

Elliptical motion in a freely swinging spherical pendulum is accompanied by an intrinsic precession[2] of angular velocity $\Omega = 3A/4l^2T$, where A is the area of the ellipse (path of the bob projected onto the horizontal plane), l is the length of the pendulum, and T is the period.[3] The sense of the precession is the same as that of the elliptical motion. The effect is strong: for example, for a pendulum of length 2 m and amplitude (center to extremity) 0.2 m, the intrinsic precession rate will be equal to the Foucault turning rate (at 40 deg latitude) if the minor diameter of the ellipse is 2.2 mm. For the error to be 20%, which might be tolerable, it would have to be less than half a millimeter, an amount scarcely perceptible to the eye.

Ellipticity in the motion of the pendulum is caused mainly by structural asymmetries, which result in the period being slightly different in one plane of oscillation than in another. The motion is that which, if allowed to continue would trace the familiar Lissajous pattern.[4] But only the early stage, the narrow ellipse, is of concern here. At that stage, the minor diameter grows at an approximately uniform rate. The rate of growth and the sense (clockwise or counterclockwise) are dependent on the azimuth. Making the mechanical system more perfect only slows the growth; it does not prevent it from reaching an intolerable magnitude eventually.[5] For the very long, museum-type pendulums the relatively smaller mechanical differences in different planes of swing, the lower frequency, and the greater tolerance for ellipticity (see formula) combine to make it possible to run for the length of vistors' hours without the development of unacceptable error in the rate of turning. But for short pendulums a device for limiting the ellipticity is imperative. The standard device is the Charron ring.[6] This is a fixed ring surrounding the suspending wire, at about a tenth of the way down from the top. The radius of the ring is such that the wire presses against it briefly, near the extremity of the swing, where the transverse velocity is maximum. The important point to note is that the negative increment in minor diameter of the ellipse, per cycle, that the ring produces is greater, the greater the minor diameter; while the positive increment per cycle due to the causes described earlier, is about constant. Therefore a crossover is reached at which the minor diameter comes to a steady value. While the steady value may be quite small, it is difficult to make it small enough for the satisfactory operation of a pendulum of much less than 2 m length. Section III deals with the irreducible residual.

III. METHOD

The method of nullifying the precession due to ellipticity to be described arose out of, and is most conveniently arranged in conjunction with, the particular type of drive system I have used on several pendulums; therefore it will be necessary to describe that system. It is similar to systems used elsewhere.[7] The bob caries a small permanent magnet (magnetic axis vertical), which passes over a circular coil as in Fig. 1(a). The magnet is long enough so the force is mainly that of the coil's field acting on the lower pole, toward or away from center. The current is constant in magnitude; its direction is reversed at the center and at each extremity of the swing, so the bob receives two outward and two inward pushes per cycle. The idea in this is to average out, in first order, any net turning that might be caused by asymmetry in the drive, for example, that due to the coil being off center.

In the course of experimenting, I set out to test how important it was to have equal inward and outward drive impulses. This was done by inserting a constant unidirectional bias current into the coil circuit. It was tried both with straight line motion and with purposely introduced elliptical motion. When the motion was elliptical, the bias changed the rate of turning. A bias of about 10 times the normal driving current and of such a sign as to produce an outward force on the bob eliminated the extra turning due to the ellipticity, leaving the Foucault turning rate. This was true
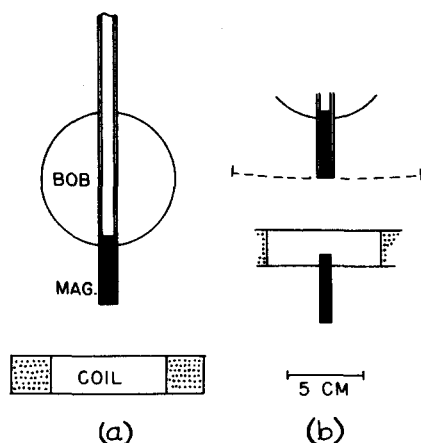
Fig. 1. (a) Arrangement of coil and magnet for driving the pendulum. (b) Detail showing the location of the lower magnet, and the arc through which the end of the upper magnet swings. The magnets are in the repelling orientation.

independently of the minor diameter of the ellipse or the sense. Since the bias required was rather large, the next move was to produce the same effect using a permanent magnet located below the bob, as shown in Fig. 1(b). Its height was easily adjusted to stop the intrinsic precession.

It is easy to see, qualitatively, how the precession is eliminated, although an analytical treatment would be difficult. Evidently, the nonlinearity in the force–displacement relation introduced by the magnets compensates that which is inherent in any pendulum having finite amplitude,[8] and which is responsible for the intrinsic precession. The compensation is not point by point, but only in its net effect; therefore it is expected to hold true only if the amplitude is constant, and if the minor diameter of the ellipse remains small compared to the separation of the poles of the two magnets. The same restriction applies to the misalignment of the axes of the magnets in the resting position: it should be small compared to the separation of the poles. With these restrictions the nullification should be independent of the minor diameter or sense of the ellipse, and that is in accord with experiment.

IV. EXPERIENCE WITH A VERY SHORT PENDULUM

I have in my office a 70-cm Foucault pendulum, mounted in the form of a "wall clock." Readings at approximately 24-hr intervals show an rms deviation from the mean of about 2%, and the mean over a week's running is within about 2% of the rate calculated for the local latitude, namely, 10.11 deg/hr. The ellipticity is only lightly restrained[9] by a ring, and at some azimuthal values it is easily visible: a millimeter or more minor diameter, an amount that, without the presence of the fixed magnet, would make the pendulum inoperable.

The optimum height of the fixed magnet was found most rapidly and sensitively by purposely increasing the minor diameter of the ellipse to, say, 5% of the distance between the adjacent poles of the two magnets. The accuracy then need not be great, for when the minor diameter is smaller, in normal operation, the uncorrected precession will be smaller in proportion. For testing, ellipticity can be increased by attaching one end of a weak spring to the suspending wire near the top and the other end to a fixed sup-

port. This changes the period slightly in one plane, and generates ellipticity when the motion is at 45 deg to that plane. The total turning rate should be measured for both clockwise and counterclockwise ellipticity, at several heights of the fixed magnet, and over the same interval of, say 5 or 10 deg. If the results are plotted, the intersection of the clockwise and counterclockwise curves will give the height of the magnet for which the intrinsic precession is reduced to zero. Setting the height in that way avoids the possible criticism that the desired Foucault rate is "fudged." At the risk of repetition, it must be emphasized that making the rate of turning invariant under reversal of the sense of the ellipticity insures that the intrinsic precession is eliminated, in a way that *does not involve even the knowledge of what the Foucault rate should be*. To be sure, one can produce any turning rate desired, by adjusting the height of the magnet, but, unless the height happens to be the unique òne found in the above way, it will give the desired rate for only one sense and minor diameter of ellipse, and therefore for only certain values of the azimuth of swing.

The construction details of the "wall clock" will be given in a separate paper. However, to put the conditions for cancellation of the precession into scale, the following figures will be useful. The length to the center of gravity (c.g.) of the bob is 70 cm, and to the end of the moving magnet 79 cm. The amplitude at the c.g. is 5.5 cm. Cancellation occurs when the distance between adjacent ends of the two magnets is 5 cm. The mass of the bob is 4.2 kg. The magnets are cylindrical, 1.75 in. long by 0.25 in. in diameter.[10] The force of repulsion between the particular pair of magnets is about 1000 dyn at 5 cm.

By way of a final remark, there seems to be no reason why the precession due to ellipticity cannot be eliminated in a pendulum of still shorter length. But there are other sources of error that, surely, at some length will become limiting. These are, mainly, friction in the gimbal, inelastic yielding of the support, air drag on the bob (especially where there is air movement in the room), and asymmetry in the drive. The phases and directions of these forces with respect to the pendulum motion can be such as to cause turning directly, rather than via the generation of ellipticity. For pendulum lengths tried so far—down to 70 cm—such effects seem to be small. The next step, from wall clock to desk clock, should be possible.

[1] I have found only two published reports of successful pendulums shorter than 2 m: that of Haym Kruglak and René Pittet at Western Michigan University, 1 m [Sci. Teach. 34 (Dec. 1979)], and that of the same authors with Stanley Steele, at the same university, 1.35 m [Sky and Telescope 60, 330-32 (Oct. 1980)]. For data on several other, longer, Foucault pendulums see Moppert and Bonwick, Ref. 7.

[2] Terms to be used: "Intrinsic precession," the rate of change of direction of the axes of the ellipse arising from the internal dynamics of the spherical pendulum, and having nothing to do with the rotation of the Earth. "Foucalt turning," the turning (in local coordinates) of the axes due to the rotation of the Earth, and which is the (siderial) angular velocity of the Earth times the sine of the latitude. "Total turning rate" is of course the sum of the above. The projection of the path of the bob on the horizontal plane is spoken of as an ellipse although it is not exactly that if it is turning, or if it is very thin as discussed in Footnote 3.

[3] For the derivation see John L. Synge and Bryon A. Griffith, *Principles of Mechanics*, 3rd ed. (McGraw-Hill, New York, 1959), p. 342; and M. G. Olsson, Am. J. Phys. 49, 531 (1981). It is to be emphasized that the formula applies to a *free* pendulum. It is not expected to hold for cases in which the motion is strongly perturbed by a device such as the

Charron ring. However, if in such cases the ellipse is narrow and the amplitude constant, the intrinsic precession might still be expected to be *proportional* to the minor diameter of the ellipse. Conceivably, a lucky combination of values of the several parameters could give zero intrinsic precession. The problem does not invite analytical treatment, because the wire slides on the ring during part of the time of contact. A further qualification should be made, and that concerns the limiting case as the minor diameter of the ellipse approaches zero. The derivation applies to a pendulum on a nonrotating planet. On the rotating Earth, the path cannot become a straight line in the limit. Consider for simplicity a pendulum at a geographic pole. If the bob comes to rest in the local frame at the extremity of the swing, which would happen under the constraint ideally exerted by a Charron ring, its path in that frame is a figure of n cusps. In the cosmic inertial frame it is an ellipse. The ellipse, of readily calculable area gives, by the intrinsic precession formula, an angular velocity of the $3\omega_e a^2/8l^2$, where a and l are the amplitude and the length of the pendulum and ω_e is the angular velocity of the Earth. This is to be subtracted from the expected Foucault turning rate. Typically, the correction is less than $\frac{1}{2}$%. If, on the other hand, the path in the cosmic frame is a straight line, it will be seen in the local frame as a rosette. The cusp and rosette paths have been treated at length by W. S. Kimball, Am. J. Phys. **13**, 271–75 (1945); and by W. B. Somerville, Quart. J. R. Astron. Soc. **13**, 40–62 (1972).

[4]This is strikingly demonstrated if the suspension is in the shape of a Y, attached to the ceiling at two points, and if the lengths are such that the periods in the planes parallel and perpendicular to the Y are different by, say, 5%. For maximum effect the pendulum should be launched at 45 deg to the plane of the Y. The sense (clockwise or counterclockwise) in which ellipticity grows alternates from quadrant to quadrant in which the motion is started. A photograph of such motion [H. R. Crane, Phys. Teach. **8**, 182 (1970)] exhibits the constant rate of growth of the minor diameter at the beginning. The path starts near the center of the photograph and is counterclockwise. The intrinsic precession, which is a smaller effect, can be seen as a rotation of the principal axes of the ellipses, between the start and the finish.

[5]If, in the 2-m pendulum used as an example, the periods in the two perpendicular directions, each at 45 deg to the direction of swing, were to differ by only a part in 10^5 (the effective lengths differing by 0.04 mm) the minor diameter would grow to 2.2 mm, enough to cancel the turning due to the Earth's rotation, in less than 200 cycles.

[6]F. Charron, Bul. Soc. Astron. France **45**, 457 (1931).

[7]Monash University in Australia [C. F. Moppert and W. J. Bonwich, Quart. J. R. Astron. Soc. **21**, 108–18 (1980)]. University of Hawaii (private communication).

[8]The problem is related to that which Christian Huygens addressed in 1658 in relation to the plane (clock) pendulum, with the object of making the period independent of amplitude. Instead of reducing the restoring force in the middle region as is done here, his method was to increase it in the outer regions. For a discussion see A. L. Rawlings, *The Science of Clocks and Watches* (Caldwell, Luling, TX, 1974).

[9]The elliptical motion is only lightly restrained for a reason. It is better to allow a small amount of residual ellipticity, whose effect is nullified by the fixed magnet, than to use a strong restraint, which may modify the turning rate. The restraint is furnished by a light weight (15 g) annular ring, (loose) located below the bob. The stem below the bob pushes it one way and then the other, at the extremities of the swing, about 1 mm. That is sufficient restraint, in view of the facts (i) that a little ellipticity is harmless, and (ii) that the supporting gimbal contains a screw by which the relative heights of the two axes of rotation is adjusted to minimize ellipticity at its source. The small energy loss at each push of the ring very effectively keeps the amplitude constant to a fraction of a millimeter. In a test in which the compensating magnet was removed, the ellipticity of about 2 mm minor diameter was maintained, the intrinsic precession agreed with the formula to within about 20%, and that was mainly the uncertainty in maintaining the minor diameter. Thus under the above set of conditions, the pendulum can be said to be nearly free.

[10]Listed in the catalog of Edmund Scientific Co., 7785 Edscorp Bldg., Barrington, NJ 08007.

PROBLEM

A primary camera lens (focal length is typically $f_1 = 50$ mm) can be converted into a telephoto lens (a lens with a longer focal length, e.g., 100 or 150 mm) by placing a secon-
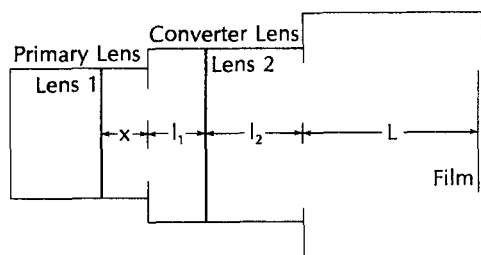

Fig. 1. Optical arrangement for teleconverter lens, primary lens, and camera body.

dary lens (teleconverter) between the primary lens and the film plane as indicated in Fig. 1.

(a) Show that the multiplication factor α (defined as the ratio of the effective focal length of the lens system to the focal length of the primary lens) is $\alpha = (L + l_2)/(L - l_1)$. Assume that the lenses are thin.

(b) Show that the focal length of the diverging converter lens is $f_2 = -(L - l_1)(L + l_2)/(l_1 + l_2)$.

(Solution is on page 1054.)

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